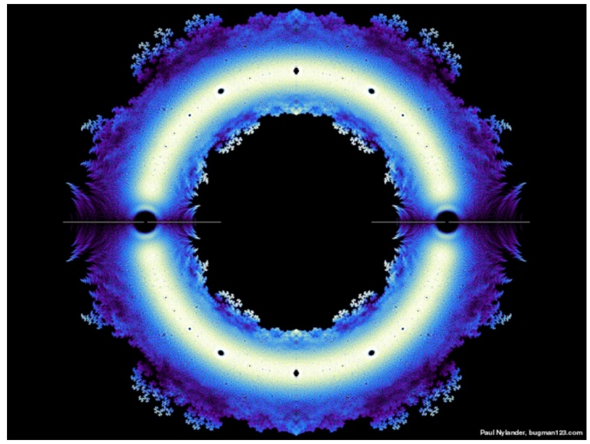
5.8: Does Every Complex Number Have a Root?

Fundamental Theorem of Algebra



The image on the right is a graph created by Paul Nylander of all the roots for all possible combinations of 18^{th} order polynomials with coefficients of ± 1.

I. Square Roots of Complex Numbers

1. Use the geometric effect of complex multiplication to describe how to calculate a square root of $z = 119 + 120i$.

2. Calculate an estimate of a square root of $119 + 120i$.

3. Every real number has two square roots. Explain why.

4. Provide a convincing argument that every complex number must also have two square roots.

5. Explain how the polynomial identity $x^2 - b = (x - \sqrt{b})(x + \sqrt{b})$ relates to the argument that every number has two square roots.

6. What is the other square root of $119 + 120i$?

EXAMPLE 1: Find the square roots of $119 + 120i$ algebraically.

Let $w = p + qi$ be the square root of $119 + 120i$. Then

$$w^2 = 119 + 120i$$

and

$$(p + qi)^2 = 119 + 120i.$$

a. Expand the left side of this equation.

b. Equate the real and imaginary parts, and solve for p and q.

c. What are the square roots of $119 + 120i$?

7. Use the method in Example 1 to find the square roots of $1 + \sqrt{3}i$.

8. Find the square roots of each complex number.
   a. $5 + 12i$

   b. $5 - 12i$

9. Show that if $p + qi$ is a square root of $z = a + bi$, then $p - qi$ is a square root of the conjugate of z, $\bar{z} = a - bi$.
   a. Explain why $(p + qi)^2 = a + bi$.

   b. What do a and b equal in terms of p and q?

   c. Calculate $(p - qi)^2$. What is the real part, and what is the imaginary part?

   d. Explain why $(p - qi)^2 = a - bi$.

Lesson Summary

The square roots of a complex number $a + bi$ will be of the form $p + qi$ and $-p - qi$ and can be found by solving the equations $p^2 - q^2 = a$ and $2pq = b$.